Backreaction of superhorizon perturbations in scalar field cosmologies

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It has been suggested that the acceleration of the Universe may be due to the backreaction of perturbations to the Friedmann-Robertson-Walker background. For a Universe dominated by cold dark matter, it is known that the backreaction of superhorizon perturbations can not drive acceleration. We extend this result to models with cold dark matter together with a scalar field. We show that the scalar field can drive acceleration only via the standard mechanism of a constant or nearly constant piece of its potential (i.e., a cosmological constant); there is no separate mechanism involving superhorizon backreaction. This rules out some models which have been proposed in the literature.

I. INTRODUCTION AND SUMMARY

The nature of dark energy is one of the most important outstanding problems in cosmology. A simple explanation is achieved by inserting a cosmological constant in Einstein's field equation. However, there are well known naturalness and fine tuning problems associated with a cosmological constant [1]. The simplest models offering a solution to the fine tuning problem are quintessence models [2]. Cosmic acceleration can also in principle be explained by modifying general relativity at large distance scales. Examples include f(R) theories [3, 4, 5, 6, 7] and the Dvali-Gabadadze-Porrati model [8, 9, 10, 11, 12, 13, 14]. For a description of more models see the review [15] and references within. The naturalness problem persists in most of these dynamical models.

It has also been suggested that the acceleration of the Universe can be explained by a purely general relativistic effect involving no new physics, the backreaction of perturbations [16, 17, 18, 19, 20]. See Refs. [17, 18, 21, 22, 23, 24] for a review of the backreaction idea. By taking a spatial average of Einstein's equations in a particular gauge, one can obtain Friedmann equations for an effective spatially averaged scale factor $a(t)$ with extra driving terms coming from backreaction [25, 26, 27, 28, 29]. These extra driving terms can in principle drive an acceleration. Although the conventional viewpoint is that the effect of backreaction is small, some have argued that it can be large enough to account for cosmic acceleration.

A problem with this theoretical approach is that the spatially averaged scale factor is not related in any simple way to quantities we observe, which average over our past light cone. See Refs. [22, 30] for a discussion of this issue.

There are two variants of the backreaction explanation. The first is that cosmic acceleration is caused by the backreaction of *superhorizon* perturbations. In particular Kolb et al [16, 17] looked at inflation-generated perturbations to a Friedmann-Robertson-Walker (FRW) universe, and claimed that at second order one could obtain a negative deceleration parameter. This claim was disproved in Refs. [31 - 33] [1].

The second variant of the backreaction explanation is that the backreaction of *subhorizon* perturbations can explain cosmic acceleration [18, 35, 36, 37, 38]. This seems unlikely but the issue has not yet been settled. We will not discuss the subhorizon backreaction issue here.

In this paper we focus on the backreaction of superhorizon perturbations in the present day Universe [2]. In particular we show that for a Universe with cold dark matter and a scalar field (as in standard quintessence models), achieving a value $q_0 \simeq -0.5$ of the deceleration parameter requires a non-zero potential. Our method of analysis is as follows [31, 32]. We compute luminosity distance as a function of redshift using Taylor series expansions, in an arbitrary Universe containing cold dark matter and a scalar field. By angle averaging we then infer the observed value of the deceleration parameter q_0. Our result shows that if such a Universe is accelerating, that acceleration must be primarily driven by the standard mechanism of a cosmological constant term in the scalar field's potential. If the potential is absent, the backreaction of superhorizon perturbation of the scalar field can not drive acceleration. In particular, second order perturbations are not sufficient to explain cosmic acceleration.

Our analysis was motivated in part by a claim by Martineau and Brandenberger [50] that the acceleration could be caused by the backreaction of superhorizon perturbations of a scalar field. These authors consider a model in which a single scalar field both drives inflation and is also present today. Modeling the scalar field per-

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[1] We also point to Ref. [34] for other arguments against cosmic acceleration caused by backreaction

[2] We note that there is also a considerable literature on superhorizon backreaction during the inflationary era, which does have a local physical effect in two scalar field inflation models [39, 40, 41, 42, 43, 44, 45, 46, 47, 48 ,49].

turbations using an effective energy momentum tensor, they argue that the effect of those perturbations can be of the right magnitude and character to cause acceleration. Our result shows that this model cannot be correct. We discuss further in Sec. III below a possible reason for our differing results.

II. COMPUTATION OF DECELERATION PARAMETER

We start by describing our theoretical framework and assumptions, which are a slightly modified version of those used in Ref. [31]. We consider the Universe in the matter dominated era, described by general relativity coupled to a pressureless fluid describing cold dark matter (we neglect baryons and radiation), together with a light scalar field. Our starting point is the assumption that backreaction is dominated by the effect of superhorizon perturbations. If this is true, then backreaction should also be present in a hypothetical, gedanken Universe in which all the perturbation modes which are subhorizon today are set to zero at early times. Generic solutions to the field equations for this gedanken Universe can be described using local Taylor series expansions rather than via perturbations of Friedmann-Robertson-Walker models, since all the fields vary on length scales or time scales of order the Hubble time or larger. This greatly simplifies the analysis.

The three equations which describe the dynamics of the gedanken universe are the following:

$$G_{\alpha\beta} = 8\pi\Bigg[\left(\rho+p\right)u_\alpha u_\beta + p g_{\alpha\beta} + \nabla_\alpha\phi\nabla_\beta\phi - \frac{1}{2}g_{\alpha\beta}\left(\nabla\phi\right)^2 - V\left(\phi\right)g_{\alpha\beta}\Bigg], \quad (1)$$

$$\Box\phi - V^{'}\left(\phi\right) = 0, \quad (2)$$

and

$$\nabla_\alpha\left[\left(\rho+p\right)u^\alpha u^\beta + p g^{\alpha\beta}\right] = 0. \quad (3)$$

Here ρ, p and u_α are the density, pressure and four velocity of matter, ϕ is the scalar field and $V\left(\phi\right)$ is its potential. Later we will specialize to cold dark matter for which $p=0$.

Next we define the specific deceleration parameter q_0 that we use. As discussed in Ref. [33], there are several different possible definitions for non-FRW cosmological models. The definition we choose matches closely with how q_0 is actually measured.

Let us start by fixing a comoving observer at point O in spacetime. We label null geodesics on $O's$ past null cone in terms of the spherical polar angles (θ,ϕ) of a local Lorentz frame at O that is comoving with the cosmological fluid. We parameterize each null geodesic in terms of an affine parameter λ and corresponding 4-momentum $\vec{k} = d/d\lambda$. For a given source S on such a null geodesic at affine parameter λ, we define the redshift as

$$1 + z\left(\theta,\phi,\lambda\right) = \frac{\vec{k}.\vec{u}|_S}{\vec{k}.\vec{u}|_O}. \quad (4)$$

The luminosity distance $\mathcal{D}_\mathcal{L}\left(\theta,\phi,\lambda\right)$ of the source S is defined in the usual way in terms of the luminosity $\left(dE/dt\right)_S$ of an assumed comoving isotropic source at S and the energy per unit area per unit time $\left(dE/dtdA\right)_O$ measured at O:

$$\left(\frac{dE}{dtdA}\right)_O = \frac{1}{4\pi{\mathcal{D}_\mathcal{L}}^2}\left(\frac{dE}{dt}\right)_S. \quad (5)$$

Assuming that the wavelength of the radiation from S is much smaller than the radius of curvature of spacetime, we can use geometric optics to compute the observed energy flux in Eq. (5) and thus the luminosity distance $\mathcal{D}_\mathcal{L}$; see, for example Ref. [51]. Finally, we can eliminate the affine parameter λ between Eqs. (4) and (5) and compute the luminosity distance as a function of spherical coordinates and redshift to obtain $\mathcal{D}_\mathcal{L} = \mathcal{D}_\mathcal{L}\left(\theta,\phi,z\right)$.

Next, to define the deceleration parameter q_0, we expand the luminosity distance in powers of redshift. The result is

$$\mathcal{D}_\mathcal{L}\left(\theta,\phi,z\right) = A\left(\theta,\phi\right)z + B\left(\theta,\phi\right)z^2 + O\left(z^3\right), \quad (6)$$

where $A\left(\theta,\phi\right)$ and $B\left(\theta,\phi\right)$ are functions that only have angular dependences. We then define the Hubble parameter H_0 and the deceleration parameter q_0 in terms of angular averages of the above functions. The standard FRW relation is

$$\mathcal{D}_\mathcal{L}\left(\theta,\phi,z\right) = H_0^{-1}z + H_0^{-1}\left(1-q_0\right)\frac{z^2}{2} + O\left(z^3\right). \quad (7)$$

Comparing the expansions (6) and (7) motivates the following definitions of H_0 and q_0:

$$H_0 \equiv \left\langle A^{-1}\right\rangle, \qquad q_0 \equiv 1 - 2H_0^{-2}\left\langle A^{-3}B\right\rangle, \quad (8)$$

where $\langle ...\rangle$ denotes an average over the angles θ and ϕ. Note that there is some ambiguity in these definitions. For example one could take $q_0 = 1 - 2\left\langle A^{-1}B\right\rangle$ instead. We choose the form (8) for computational convenience, and we will argue below that the differences are unimportant.

We next explicitly evaluate the expressions (8) for H_0 and q_0. We consider generic solutions to the equations (1) - (3), described by local Taylor series expansions [52]

about the observer O. The expressions for the functions A and B were computed in Ref. [31], and are

$$A(\theta,\phi) = \frac{1}{\left(\nabla^\alpha u^\beta\right) k_\alpha k_\beta}, \tag{9}$$

$$B(\theta,\phi) = \frac{2}{\left(\nabla^\alpha u^\beta\right) k_\alpha k_\beta} + \frac{\left(\nabla^\alpha \nabla^\beta u^\gamma\right) k_\alpha k_\beta k_\gamma}{2\left[\left(\nabla^\alpha u^\beta\right) k_\alpha k_\beta\right]^3}, \tag{10}$$

where all quantities on the right hand sides are evaluated at O. By inserting the expression for $A(\theta,\phi)$ into the definition (8) of H_0 and evaluating the angular average, we obtain

$$H_0 = \frac{1}{3}\Theta, \tag{11}$$

where

$$\Theta = \nabla_\alpha u^\alpha \tag{12}$$

is the expansion of the cosmological fluid. This is the same result as was obtained in Ref. [31].

Next, we insert the expressions (9) and (10) for $A(\theta,\phi)$ and $B(\theta,\phi)$ into (8) to obtain

$$\frac{1}{2} H_0^2 \left(1 - q_0\right) = 2\left\langle \left[\left(\nabla^\alpha u^\beta\right) k_\alpha k_\beta\right]^2 \right\rangle$$

$$+ \frac{1}{2}\left\langle \left(\nabla^\alpha \nabla^\beta u^\gamma\right) k_\alpha k_\beta k_\gamma \right\rangle. \tag{13}$$

We now evaluate these angular averages using the same techniques as in Ref. [31]. The only difference from the computation of Ref. [31] arises when we eliminate a factor of the Ricci tensor $R_{\alpha\beta}$ using the field equations. Here that elimination generates extra terms involving the scalar field, from the equation of motion (1). The final result is

$$q_0 = \frac{4\pi}{3H_0^2}\left[\rho + 3p - V(\phi) + 2\nabla_\alpha\phi\nabla_\beta\phi u^\alpha u^\beta\right]\Big|_O$$

$$+ \frac{1}{3H_0^2}\left(a_\alpha a^\alpha + \frac{7}{5}\sigma_{\alpha\beta}\sigma^{\alpha\beta} - w_{\alpha\beta}w^{\alpha\beta} - 2\nabla_\alpha a^\alpha\right)\Bigg|_O. \tag{14}$$

Here $\sigma^{\alpha\beta}$, $w^{\alpha\beta}$ and a^α are the shear, vorticity and 4-acceleration of the fluid, defined by

$$\nabla_\alpha u_\beta = \frac{1}{3}\Theta g_{\alpha\beta} + \sigma_{\alpha\beta} + w_{\alpha\beta} - a_\beta u_\alpha, \tag{15}$$

with

$$\sigma_{\alpha\beta} = \sigma_{\beta\alpha} \text{ and } w_{\alpha\beta} = -w_{\beta\alpha}. \tag{16}$$

III. DISCUSSION

We now specialize our result (14) to a pressureless fluid. The 4-acceleration then vanishes and we have

$$q_0 = \frac{4\pi G}{3H_0^2}\left[\rho - V(\phi) + 2\nabla_\alpha\phi\nabla_\beta\phi u^\alpha u^\beta\right]\Big|_O$$

$$+ \frac{1}{3H_0^2}\left(\frac{7}{5}\sigma_{\alpha\beta}\sigma^{\alpha\beta} - w_{\alpha\beta}w^{\alpha\beta}\right)\Bigg|_O. \tag{17}$$

We now argue that the only way to achieve $q_0 \approx -0.5$, as required by observations, is to have $V(\phi_0)$ be large and positive, where ϕ_0 is the value of ϕ evaluated at the observer.

We can estimate the terms $\sigma_{\alpha\beta}\sigma^{\alpha\beta}$ and $w_{\alpha\beta}w^{\alpha\beta}$ in Eq. (17) to be $\sim (\delta v)^2/l^2$, where δv is the typical scale of peculiar velocity perturbations, and l is the scale over which the velocity varies. Since we have assumed that subhorizon perturbations are absent we have $l \gtrsim H_0^{-1}$. This implies that the contributions from these terms are of order $\delta q_0 \sim (\delta v)^2 \simeq 10^{-4}$. Since the measured value of q_0 is $q_0 \sim -0.5$, these terms cannot contribute significantly to the deceleration parameter.

In the first term in Eq. (17), the quantities ρ and $2\nabla_\alpha\phi\nabla_\beta\phi u^\alpha u^\beta$ are always positive (the second term is the square of $\sqrt{2}\nabla\phi_\alpha u^\alpha$). This means that the potential term has to be larger than the sum of these two terms to get negative deceleration. Thus, a large negative deceleration must come primarily from the potential.

We note that this result differs from that obtained by Martineau and Brandenberger in Ref. [50], who found that the backreaction of superhorizon perturbations *could* drive cosmic acceleration via a mechanism not involving the potential. A possible reason for the difference is the fact that different measures of cosmic acceleration are used in the two different analyses. The authors of Ref. [50] use a measure that is based on averages over a spatial slice at a given instant of time (which is inherently gauge dependent). We use a different measure which is essentially an average over the past light cone of the observer, and is gauge independent. Moreover, our measure corresponds more closely to the actual deceleration parameter that has been measured.

Finally, we note that the specific choices of angle averaging prescriptions in the definitions (8) of the Hubble parameter and deceleration parameter are not unique. However, as was argued in Ref. [31], the change that results from adopting other definitions is negligible. For example, one could consider the alternative definition

$$q_0 \equiv 1 - 2H_0\langle B\rangle \tag{18}$$

of the deceleration parameter. In Ref. [31] it was shown that this alters the final result (17) in three ways: (i) Changing the numerical coefficients of the shear squared and vorticity squared terms by an amount of order unity,

which does not affect our conclusions; (ii) The addition of new terms that are comparable to the shear squared and vorticity squared terms; and (iii) The addition of new terms that are suppressed compared to the shear squared and vorticity squared terms by one or more powers of the dimensionless ratio (non-isotropic part of $\nabla_\alpha u_\beta$)/(isotropic part of $\nabla_\alpha u_\beta$). This dimensionless ratio is constrained observationally to be small compared to unity, since peculiar velocities on Hubble scales today are small. The same arguments continue to apply in the present context, since the scalar field dependent terms in (17) are unchanged by the change in definition of q_0.

IV. CONCLUSION

The backreaction of perturbations is sometimes considered to be a candidate for explaining cosmic acceleration [53]. Many techniques have been developed to explore the effects of backreaction. In this paper, we computed the deceleration parameter measured by comoving observers in a hypothetical universe with all perturbation modes which are subhorizon today set to zero at early times. We considered a universe containing cold dark matter and a minimally coupled scalar field. We showed that one can obtain a large negative value of the deceleration parameter in this context only if the deceleration is primarily produced by the scalar field's potential.

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